Q1

1

Solve, in the range $-\pi < \theta \leq \pi$, the equation

$$\frac{\sec\theta \cot\theta}{\operatorname{cosec}\theta \tan\theta} = -\sqrt{3}.$$

[5]

$\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$ $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\tan = \frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta} \cdot \frac{1}{\tan\theta} \div \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta\tan\theta} \cdot \frac{\cos\theta}{1}$

$\frac{1}{\tan\theta} = \cot\theta = -\sqrt{3}$

$\tan\theta = -\frac{1}{\sqrt{3}}$ OR $-\frac{\sqrt{3}}{3}$

$\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{1}{6}\pi$ $+\pi$

$\theta = -\frac{1}{6}\pi, \frac{5}{6}\pi$

Q2

2

Solve, in the range $0 \leq \theta \leq 2\pi$, the equation

$$6\sec\theta + \frac{2\sqrt{3}}{\sec\theta} = -3 - 4\sqrt{3}.$$

Leaving your answers as exact values.

[6]

MULTIPLY FACTORISE SOLVE

$6\sec^2\theta + 2\sqrt{3} = (-3 - 4\sqrt{3})\sec\theta$

$6\sec^2\theta + (3 + 4\sqrt{3})\sec\theta + 2\sqrt{3} = 0$

$(3\sec\theta + 2\sqrt{3})(2\sec\theta + 1) = 0$

SOLVE

$3\sec\theta + 2\sqrt{3} = 0$

$\sec\theta = -\frac{2\sqrt{3}}{3}$

$\frac{1}{\cos\theta} = -\frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{-3}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}$

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5}{6}\pi$

SYMMETRICAL AROUND π

$\theta = \frac{7}{6}\pi$

$2\sec\theta + 1 = 0$

$\sec\theta = -\frac{1}{2}$

$\sec\theta = k$

NO SOLUTIONS FOR $-1 < k < 1$

$\theta = \frac{5}{6}\pi, \frac{7}{6}\pi$

3

Q3

3

Using the double angle formulae $\sin 2A \equiv 2\sin A\cos A$ and $\cos 2A \equiv \cos^2 A - \sin^2 A$, find the solutions to the equation

$$(\operatorname{cosec} x - \sec x)\left(\frac{1}{\sec x} + \frac{1}{\operatorname{cosec} x}\right) = \cot 2x + 3$$

in the range $-\pi < x \leq \pi$. Give your answers correct to 3 significant figures.

[6]

EXPAND LHS

$\frac{\operatorname{Cosec} x}{\sec x} + 1 + 1 - \frac{\sec x}{\operatorname{cosec} x}$

$\operatorname{Cosec} x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$

$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$

$\times \frac{2}{2}$ $\quad \frac{2\cos^2 x - \sin^2 x}{2\sin x\cos x} = \frac{2\cos 2x}{\sin 2x}$ DOUBLE ANGLE

$\cot x = \frac{\cos x}{\sin x}$ $\quad$ LHS $= 2\cot 2x$

$2\cot 2x = \cot 2x + 3$

$\cot 2x = 3$

SOLVE

$\frac{1}{\tan 2x} = 3$

$\tan 2x = \frac{1}{3}$

LET $z = 2x$ $\quad$ RANGE $-2\pi < 2x < 2\pi$

$\tan^{-1}\left(\frac{1}{3}\right) = 0.32175$

$-2\pi, -\pi, +\pi$

$z = -5.96, -2.82, 0.322, 3.46$

$\div 2$

$x = -2.98, -1.41, 0.161, 1.73$ (3sf)

Q4

4

Solve, in the range $0 \leq x \leq 2\pi$, the equation

$$3\cot^2 x - 4\sqrt{3} = (6 - 2\sqrt{3})\operatorname{cosec} x - 3.$$

Leaving your answers as exact values.

[6]

$1 + \cot^2 x \equiv \operatorname{cosec}^2 x$

$\cot^2 x \equiv \operatorname{Cosec}^2 x - 1$

$3(\operatorname{Cosec}^2 x - 1) - 4\sqrt{3} = (6 - 2\sqrt{3})\operatorname{Cosec} x - 3$

$3\operatorname{Cosec}^2 x - 3 + (2\sqrt{3} - 6)\operatorname{Cosec} x + 3 - 4\sqrt{3} = 0$

$3\operatorname{Cosec}^2 x + (2\sqrt{3} - 6)\operatorname{Cosec} x - 4\sqrt{3} = 0$

$(3\operatorname{Cosec} x + 2\sqrt{3})(\operatorname{Cosec} x - 2) = 0$

SOLVE

$3\operatorname{Cosec} x + 2\sqrt{3} = 0$

$\operatorname{Cosec} x = -\frac{2\sqrt{3}}{3}$

$\frac{1}{\sin x} = -\frac{2\sqrt{3}}{3}$

$\sin x = \frac{-3}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{1}{3}\pi$ OUT OF RANGE

$\pi + \frac{1}{3}\pi = \frac{4}{3}\pi$

$2\pi - \frac{1}{3}\pi = \frac{5}{3}\pi$

$\operatorname{Cosec} x - 2 = 0$

$\operatorname{Cosec} x = 2$

$\frac{1}{\sin x} = 2$

$\sin x = \frac{1}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{1}{6}\pi$

$\pi - \frac{1}{6}\pi = \frac{5}{6}\pi$

$x = \frac{\pi}{6}, \frac{4}{3}\pi, \frac{5}{3}\pi, \frac{5}{6}\pi$

Q5a

5a

(a) Sketch, in the interval $-2\pi \le \theta \le \pi$, the graph of $y = 2 + 3\sec\left(\theta + \frac{\pi}{2}\right)$, include asymptotes and label the coordinates of all maximum and minimum points.

[3]

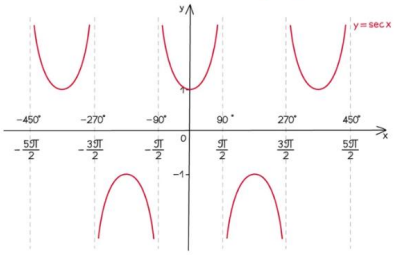
(b) Deduce the maximum and minimum values of $\frac{1}{2+3\sec\left(\theta+\frac{\pi}{2}\right)}$.

[4]

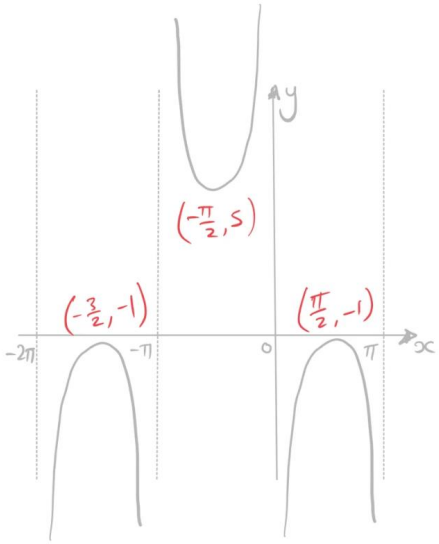
TRANSLATE $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$

VERTICAL STRETCH SF 3

TRANSLATE $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$



a)



Q5b

5b

(a) Sketch, in the interval $-2\pi \le \theta \le \pi$, the graph of $y = 2 + 3\sec\left(\theta + \frac{\pi}{2}\right)$, include asymptotes and label the coordinates of all maximum and minimum points.

[3]

(b) Deduce the maximum and minimum values of $\frac{1}{2+3\sec\left(\theta+\frac{\pi}{2}\right)}$.

[4]

b) y VALUES

MAX $= \frac{1}{\text{MIN}}$ POSITIVE (a)

$\text{MAX} = \frac{1}{5}$

MIN $= \frac{1}{\text{MAX}}$ NEGATIVE (a)

$\text{MIN} = \frac{1}{-1} = -1$

$\text{MAX} = \frac{1}{5}$, $\text{MIN} = -1$